LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – STATISTICS

FIRST SEMESTER – NOVEMBER 2009

ST 1814 / 1809 - MEASURE AND PROBABILITY

Date & Time: 06/11/2009 / 1:00 - 4:00 Dept. No. Max. : 100 Marks

SECTION-A (10x2=20 marks)
Answer All questions:

1) Define the limit of a sequence $\{A_n\}$ of sets.
2) For a sequence $\{A_n\}$ of sets, if $A_n \to A$, show that $A_n^c \to A^c$.
3) Show that a σ- field is monotone field .
4) What is the minimal σ- field containing a given class of sets?
5) If μ is measure, show that $\mu(\bigcup_{n=1}^{\infty} A_n) \leq \sum_{n=1}^{\infty} \mu(A_n)$.
6) Calculate E(X), if X has a distribution function F(x), where

$$F(x) = \begin{cases} 0 & \text{if } x<0 \\ x/2 & \text{if } 0 \leq x<1 \\ 1 & \text{if } x \geq 1. \end{cases}$$

7) If $E(Y \mid X) = \alpha X+\beta$ and X has standard normal distribution, evaluate E(Y).
8) Show that $\phi(t) = \dfrac{e^{-|t|} + e^{-\frac{t2}{2}}}{2}$ is a characteristic function of a random variable.
9) A random variable X has characteristic function

$$\phi(t) = \begin{cases} \dfrac{\sin t}{t}, & t \neq 0 \\ 1, & t = 0 \end{cases}$$

10) State Lindeberg – Feller central limit theorem

SECTION-B (5x8=40 marks)
Answer any FIVE questions:

11) Show that the inverse image of a σ- field is a σ- field.

12) (a) Define (i) a finitely additive and
 (ii) a countably additive set functions.

 (b) Let Ω = {-3,-1,0,1,3} and for $A \subset \Omega$, let $\lambda(A) = \sum_{k \in A} k$ with $\lambda(\phi) = 0$. Show that λ is countably additive. If λ' = min(λ,0), show that λ' is not even finitely additive.

13) If X is a non-negative measurable function, show that there exits a non-decreasing sequence of non-negative simple functions $\{X_n, n\geq 1\}$ converging everywhere to X.

14) If X and Yare integrable simple functions, prove that

$$\int_\Omega (X+Y)\, d\mu = \int_\Omega X\, d\mu + \int_\Omega Y\, d\mu$$

15) Define the distribution function of a random variable X. State and establish its defining properties.

16) State and prove Borel zero –one law.

17) (a) Define(i) convergence in quadratic mean .
(ii) almost sure convergence for a sequence of random variables.
(b) Show that convergence in quadratic mean implies convergence in probability.

18) Let $\{ X_n \}$ be a sequence of independent random variables with common frequency function $f(x) = 1/x^2$, $x > 1$. Show that X_n /n does not converge to zero with probability one.

SECTION-C (2x20= 40 marks)

Answer any TWO questions.

19) (a) Show that a σ-ring is closed under countable intersection. (4marks)
(b) Show that the minimal σ-field containing the class of all open intervals (a,b) is the minimal σ-field containing the class of all closed intervals [a,b]. (8marks)
(c) Show that σ-field is a field. Is the converse true? Justify. (8marks)

20) (a) Define (i) measurable function. (ii) Borel function.
Show that the limits of measurable functions are also measurable functions. (10marks)
(b) State and prove Monotone convergence theorem. (10marks)

21) (a) Show that the probability distribution of a random variable is determined by its distribution function. (4marks)
(b) Explain the independence of two random variables X and Y. If X and Y are independent, then show that X^2 and Y^2 are independent.What about the converse? (8marks)
(c) Find var(Y),if the conditional characteristic function of Y given X = x is $\left(1+\frac{t2}{x}\right)^{-1}$ and X has frequency function

$$f(x) = \begin{cases} 1/x^2, & \text{for } x \geq 1 \\ 0, & \text{otherwise} \end{cases}$$

22) (a) State and prove Kolmogorov three series theorem for almost sure convergence of the series ΣX_n of independent random variables. (12marks)
(b) Let $\{ X_n , n \geq 1\}$ be a sequence of independent random variables such that X_n has
uniform distribution on $\left(-\frac{1}{n},\frac{1}{n}\right)$, $n \geq 1$.
Examine the series $\sum_{n=1}^{\infty} Xn$ for almost sure convergence. (8marks)

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